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# THE SPACE-DEVELOPED DYNAMIC VERTICAL CUTOFF RIGIDITY MODEL AND ITS APPLICABILITY TO AIRCRAFT RADIATION DOSE

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## ABSTRACT

We have developed a dynamic geomagnetic vertical cutoff rigidity model that predicts the energetic charged particle transmission through the magnetosphere. Initially developed for space applications, we demonstrate the applicability of this library of cutoff rigidity models for computing aircraft radiation dose. The world grids of vertical cutoff rigidities were obtained by particle trajectory tracing in a magnetospheric model. This reference set of world grids of vertical cutoff rigidities calculated for satellite altitudes covers all magnetic activity levels from super quiet to extremely disturbed (i.e., Kp indices ranging from 0 to 9<sup>+</sup>) for every three hours in universal time. We utilize the McIlwain "L" parameter as the basis of the interpolation technique to reduce these initial satellite altitude vertical cutoff rigidities to cutoff rigidity values at aircraft altitudes. Published by Elsevier Ltd on behalf of COSPAR.

## INTRODUCTION

We have a complete set of world grids of vertical cutoff rigidities each 5° in latitude and 5° in longitude for a spacecraft orbiting at 450 km. These vertical cutoff rigidities were obtained by particle trajectory tracing in model magnetospheres and include values for all magnetic activity levels from super quiet to extremely disturbed (i.e., Kp indices ranging from 0 to 9<sup>+</sup>) for every three hours in universal time. We used the Tsyganenko (1989) magnetospheric field model combined with the International Geomagnetic Reference Field for Epoch 1995.0 (Sabaka et al., 1997). The Tsyganenko (1989) magnetospheric field model describes the magnetospheric field topologies for Kp<sup>\*\*</sup> magnetic indices from 0 to 5. The Boberg et al., (1995) extension was used to describe the magnetospheric fields for magnetic activity levels exceeding Kp values of 5. For convenience we have labeled these as Kp 6 through 10 in Dst<sup>†</sup> increments of -100 nT.

## CUTOFF RIGIDITY DETERMINATION PROCEDURE

Cosmic ray trajectory calculations were initiated in the vertical direction from a distance of 6821.2 km from the geocentric (450 km altitude above the average earth radius of 6371.2 km). We utilized the Bulirsch-Stoer numerical integration technique (Press et al., 1989); each step length was about 1% of the gyro-distance. The "sensible" atmosphere of the earth was considered to extend 20 km above the international reference ellipsoid.

\* Rigidity is momentum per unit charge and is a canonical unit that is especially useful in characterizing charged particle access in magnetic fields. All particles having the same magnetic rigidity will have identical trajectories in the magnetic field, independent of elemental or isotopic composition, particle mass or atomic charge.

\*\* Kp is a quasi-logarithmic magnetic activity index that ranges from zero to 9 in major increments. There are 3 sub-increment levels (-, 0, and +) within each major increment.

† Dst is an index of the magnetic fields generated by during magnetic storms, measured in units of Nt at the earth's equator.

soid, and any trajectory path that came lower than this distance was considered to be re-entrant and hence forbidden. Geomagnetic cutoff rigidities are determined by calculating charged particle trajectories at discrete rigidity intervals starting with a rigidity value high above the highest possible cutoff and decreasing the rigidity to a value that satisfied our criteria that the lowest allowed trajectory had been calculated. As the calculations progress down through the rigidity spectrum, the results change from the easily allowed orbits to a complex structure of allowed, forbidden, and quasi-trapped orbits (loosely called penumbra) and finally to a set of rigidities where all trajectories intersect the solid earth. Rigidity interval spacing of 0.01 GV was used throughout the cosmic ray penumbra. As a result of these trajectory calculations we determined the calculated upper cutoff rigidity ( $R_U$ ) which is the rigidity value of the highest allowed/forbidden pair of adjacent cosmic ray trajectories, the calculated lowest cutoff rigidity ( $R_L$ ) which is the rigidity value of the lowest allowed/forbidden pair of adjacent cosmic ray trajectories, and an "effective cutoff rigidity" ( $R_C$ ) found by summing the allowed orbits through the penumbra. See Cooke et al. (1991) for definitions of cosmic ray cutoffs.

Use of these updated three-hour UT interval geomagnetic cutoff rigidity models results in better fits to experimental data than the previous values of the six-hour UT world grids (Smart et al., 1999a,b,c). A comparison of the components of this dynamic cutoff rigidity model with vertical cutoff rigidities calculated using only an internal geomagnetic field for the same altitude (Smart and Shea, 1997) show that these magnetospheric cutoff rigidity values are consistently lower in magnitude, even for the  $K_p=0$  case.

Rigidity (momentum per unit charge) is not the most convenient unit for use with energetic particle data since most energetic particle measurements are in units of energy. For comparison purposes, we have selected the invariant latitude<sup>†</sup> calculated from the internal geomagnetic field as a common parameter. We have interpolated through our world grids of vertical geomagnetic cutoff rigidities for each magnetic activity level to determine proton cutoff energy contours as a function of invariant latitude and obtained an average invariant latitude for each energy. The invariant latitudes of the proton cutoff for all the magnetic activity levels are illustrated in Figure 1. These curves indicate an almost linear relation between the proton cutoff energy with magnetic latitude in the range from about 10 MeV to a few hundred MeV. We note that the change of proton cutoff energy with  $K_p$  is relatively uniform over the range of the original Tsyganenko (1989) model ( $K_p$  values 0 through 5), but the cutoff changes introduced by the Boberg et al. (1995) extension are non-linear with the Dst increment.

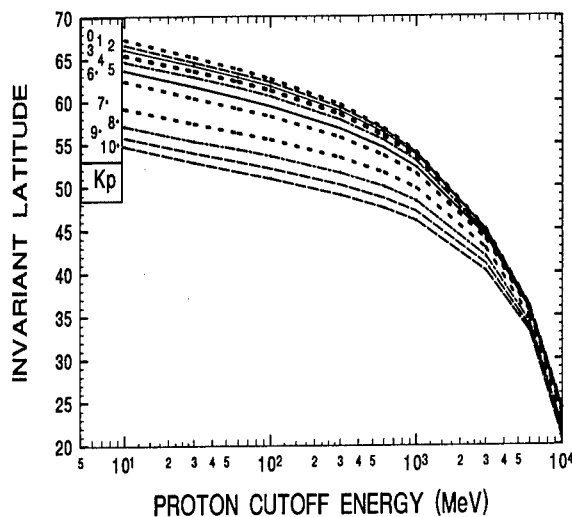


Fig. 1. The invariant magnetic latitude of the proton effective vertical cutoff energy at 450 km for all integer magnetic activity levels.

Geomagnetic cutoffs are angle-dependant. Using Störmer (1955) theory we can extrapolate from the vertical cutoff rigidity to the cutoff at other azimuth and zenith angles by the following equations:

$$R_v = V_K \cos^4 \lambda. \quad (1)$$

$$R_\alpha = 4R_v \setminus \{ [1 + (1 - \sin \epsilon \sin \phi \cos^3 \lambda)^{1/2}]^2 \}. \quad (2)$$

$R_v$  is the vertical cutoff rigidity,  $V_K$  is the constant,  $R_\alpha$  is the cutoff rigidity in a specific angular direction,  $\lambda$  is appropriate magnetic latitude,  $\epsilon$  is the angle from the zenith direction, and  $\phi$  is the azimuthal angle measured clockwise from magnetic north. The angular geomagnetic cutoffs are lowest from the magnetic west and maximum from the magnetic east. In Störmer theory, the cutoffs in the magnetic north and south directions are the same as the vertical cutoff.

<sup>†</sup> Invariant latitude is a development from magnetospheric physics. It is a magnetic latitude that accounts for the effective magnetic center displacement from the geocenter, the orientation of the magnetic field axis, and the higher order terms in the magnetic field.

## ALTITUDE INTERPOLATION

The basis of the interpolation technique is the fact that the McIlwain "L" parameter (McIlwain, 1961) can be utilized in equation (1) for cosine squared of the magnetic latitude. Then, the vertical cutoff rigidity ( $R_c$ ) equation can be written in the form

$$R_c = V_k / L^2. \quad (3)$$

The latitude and longitude of the location for which we want to determine the cutoff rigidity will be contained in a specific "box" on the 5-degree by 5-degree world grid. Each "box" contains the  $V_k$  and  $L$  parameters for each corner. The orientation of the specified latitude and longitude position within the grid box is then determined. We use the  $L$  value of the latitude and longitude (at 450 km) and the  $V_k$  constants for the above cutoff equation at each "box corner", to interpolate a cutoff value for the coordinate (latitude and longitude) in the "box". A value is obtained for the upper cutoff,  $V_{k(RU)}$ , the lower cutoff,  $V_{k(RL)}$ , and the effective cutoff,  $V_{k(RC)}$ . Then the McIlwain "L" value is computed at the specific coordinate and for both altitudes with "L" interpolation used to extrapolate from the values at 450 km altitude to the specified altitude.

## COMPARISON WITH SPACECRAFT PROTON CUTOFF DATA

In order to test the accuracy of the interpolated cutoff values, we compare our computed invariant latitude of the solar proton cutoff with the measured cutoff latitudes observed by the SAMPEX spacecraft (Leske et al., 1997). The solid dark line at 67.5 degrees in Figure 2 indicates the invariant cutoff latitude predicted from use of the internal IGRF magnetic field model (Smart and Shea, 1997). The gray squares indicate the computed cutoff invariant latitude for the center energy of the 29-64 MeV detector. The cutoff latitude for each Kp value is indicated on the left of the figure and the integer Kp values are on the right. The solid diamonds are the cutoff latitudes of Leske et al. (1997). This figure shows that the interpolated and measured cutoff latitudes exhibit the same general trend, for both magnetically quiet and active times.

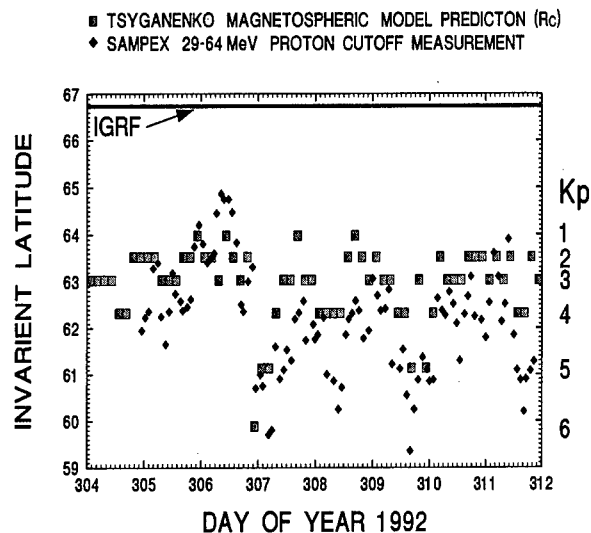


Fig. 2. Comparison of the calculated and measured invariant latitudes of the proton cutoff energy as a function of the Kp index.

## COMPARISON WITH SPACECRAFT DOSIMETRY DATA

The STS28 space shuttle flight in August 1989 was in a 57° inclination orbit and carried a dosimeter that provided data on the large solar proton event sequence that began on 12 August 1989. The flight was completed on 13 August with the shuttle landing shortly after 13 UT. We used the dosimetry data acquired during the actual space flight as a method of evaluating the accuracy of the dynamic cutoff rigidity model. During the time the vehicle encountered solar protons, the geomagnetic activity level as quantified by the Kp index varied between 1 and 3. We used our dynamic cutoff rigidity model to predict the solar proton transmission through the magnetosphere to the position of the space shuttle (latitude, longitude and altitude) each minute of the shuttle flight during its encounter with the solar proton event. The solar particle flux predicted to arrive to the skin of the space shuttle was used as input to the NASA/JSC PDOSE code to compute the radiation dose rate. A more detailed description of this radiation dose rate calculation method is given by Golightly and Weyland (1997). The general procedure was:

1. Determine (by interpolation in the appropriate  $5^\circ \times 5^\circ$  world grid of cutoff rigidities) the vertical cutoff proton energy at the spacecraft position for each minute of the solar proton event encounter.
2. Model the GOES measured solar proton flux as the differential flux impacting the magnetosphere. We used a three-section fit of the solar proton energy spectrum: an exponential between 30 and 50 MeV, another exponential between 50 and 60 MeV, and another exponential between 60 and 100 MeV. The fit between 60 and 100 MeV was used to extrapolate to higher energies.
3. Determine the solar proton flux at energies exceeding the proton cutoff energy at the spacecraft position.
4. Reduce the free space omni-directional flux to account for "earth shadowing" in low-earth orbit.
5. Attenuate the incident solar proton through the spacecraft mass distribution.
6. Calculate the dose rate from the attenuated solar proton flux spectrum penetrating to the dosimeter location.

### Comparisons Using Cutoff Parameters

The STS28 dose rate data set also provided an opportunity to check the applicability of the various cutoff rigidity parameters such as  $R_U$ ,  $R_L$ , and  $R_C$ . We found that truncating the Kp magnetic activity index (i.e. reducing the + sub increment) results in a cutoff value at the spacecraft position that is too high and the solar particle flux predicted to be allowed at the spacecraft was not sufficient to reproduce the dose measurement. It is necessary to "round up" the Kp index in order to obtain a cutoff value that predicts a dose rate that is consistent with the measured dose rate.

Evaluation of the solar proton flux allowed to the spacecraft using each of the of three cutoff rigidity parameters resulted in the following:

1. Use of the upper cutoff rigidity,  $R_U$ , resulted in an underprediction of the dose rate.
2. Use of the lower cutoff rigidity,  $R_L$ , resulted in an overprediction of the dose rate.
3. Use of the effective cutoff rigidity,  $R_C$ , resulted in a slight systematic underprediction of the dose rate.
4. Using all of the solar particle flux down to  $R_U$  and then using the average transparency of the penumbra to specify the solar particle flux transmitted through the penumbra between  $R_U$  and  $R_L$  resulted in a systematically better correspondence between the computed and measured dose rate. This average transparency, a function of the cutoff rigidity, is shown in Figure 3.

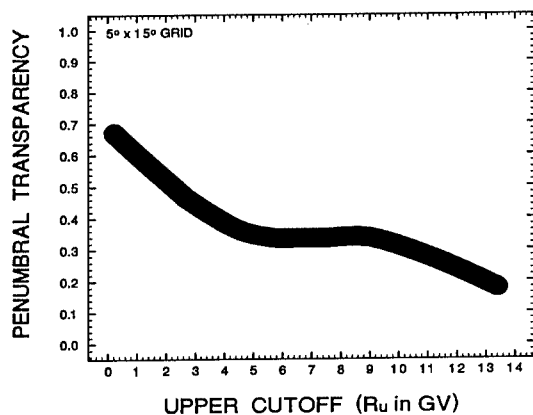


Fig. 3. Average transparency of the penumbra.

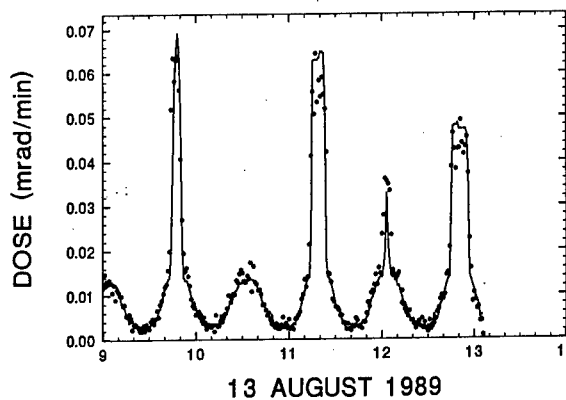


Fig. 4. Comparison of computed (solid line) and observed (dots) radiation dose rate during the 13 August 1989 solar proton event.

It was also necessary to include the effect of the lower cutoffs in the western direction to reproduce the observed radiation dose intensity/time profile. The allowed solid angle of particle access was divided into five equal solid angle segments, vertical, north, south, east and west. The particle flux allowed through the geomagnetic cutoff for each segment was summed to find the allowed particle flux over the solid angle of particle access. Employing the dynamic cutoff rigidity model with a proper selection of the magnetic activity index resulted in a minute-to-minute correspondence between the time periods of computed radiation dose rate due to

solar protons being allowed through the magnetosphere to the position of the space shuttle and the measured dose rate in the vehicle, even for very small doses. Employing the solar proton flux spectral and the appropriate Kp magnetic activity indices enabled computation of the solar particle dose rate for each minute of the STS28 flight. The comparison between the computed and measured dose rate is given in Figure 4. A more detailed description of these results are given by Smart and Shea (2002) and Smart et al. (2003).

### EXTRAPOLATION TO THE EARTH'S ATMOSPHERE

Since we have a method of extrapolating vertical cutoff rigidities to various spacecraft altitudes, it seems apparent that we can use the same procedure to extrapolate to the atmosphere. We will demonstrate this by calculating vertical cutoff rigidities for the Concorde flight from London Heathrow Airport (LHR) to New York Kennedy airport (JFK) that occurred on 24 October 1989. During this flight high-energy solar cosmic rays (illustrated in Figure 5) provided radiation exposure in excess of the background cosmic radiation. During the flight time the Kp magnetic index was 4<sup>-</sup>. The aircraft position coordinates provided by Dyer (2002) were used to calculate the vertical cutoff rigidity and the vertical proton cutoff energy, along the flight path.

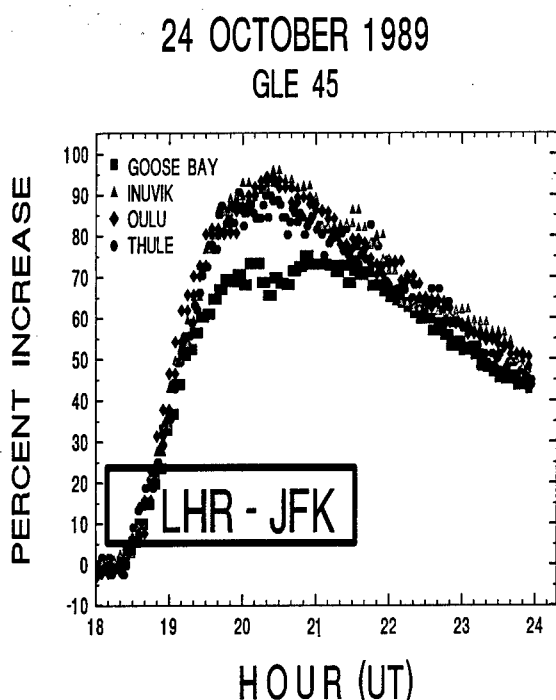


Fig. 5. The high-energy solar cosmic ray event of 24 October 1982 as observed by high latitude neutron monitors. Oulu, Finland, Goose Bay, Canada, and Thule, Greenland are positioned near the boundaries of the North Atlantic. The time of the Concorde flight is indicated by the box.

Since this was an interval of disturbed geomagnetic conditions there was a reduction of the geomagnetic cutoff below the normal values. Figure 6 illustrates the normal quiescent cutoff rigidity values that would be experienced along this flight path together with the values calculated for the actual geomagnetic conditions for this specific flight. (Western hemisphere longitudes are designated as negative in this Figure.) Further analysis of these cutoff rigidity data and the corresponding radiation dose experienced at flight altitude has been done by Dyer et al., (2003).

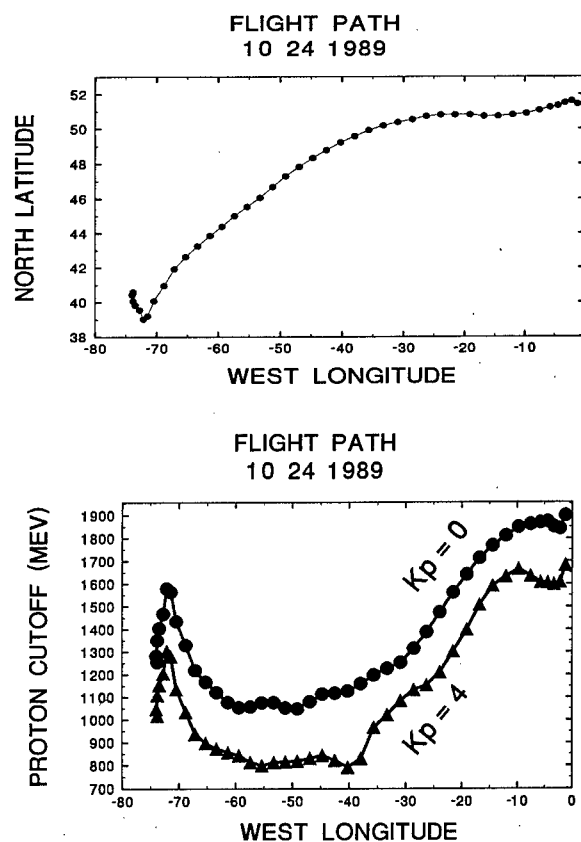


Fig. 6. Top: Path of the LHR-JFK Concorde flight on 24 October 1989. Bottom: Calculated vertical proton cutoff energy along the flight path for quiet ( $K_p=0$ ) and actual  $K_p=4$  magnetic activity.

## FINAL REMARKS

We have developed a dynamic geomagnetic vertical cutoff rigidity model that predicts the energetic charged particle transmission through the magnetosphere. This reference set of world grids covers all magnetic activity levels from super quiet to extremely disturbed (i.e., Kp indices ranging from 0 to 9+) for every three hours in universal time. These world grids of vertical cutoff rigidities were obtained by particle trajectory tracing in model magnetospheres. Testing of the uses of these interpolated cutoff rigidities compares very favorably with actual spacecraft measurements of the cutoff latitude. Use of these cutoff rigidity models for computing radiation dose resulted in a one-to-one correspondence between the portion of the orbit predicted to be subjected to solar protons and the portion of the orbit where the radiation dose can be attributed to solar particles. Although developed for space applications, we demonstrate the applicability of this library of cutoff rigidity models for computing the cutoff rigidity along aircraft routes.

## ACKNOWLEDGMENTS

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